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Fuzzy mathematical modeling in the Gherin Analysis

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Abstract

The main aim of this paper is to discuss the basic ideas and ideas of the purported" Fluffy Mathematics" and to give a brief study of the history and of certain patterns in late improvement of mathematics and its applications with regards to fluffy sets. As a potential peruser we imagine a mathematician, who is not working in the field of "fluffy mathematics", however wishes to have some idea about this immense field in present day science.

***Keywords**: fuzzy set, fuzzy logic, fuzzy real number, fuzzy topology.*

Introduction

Since the inception of the idea of a theoretical set by Georg Cantor toward the finish of the nineteenth hundred years, the sets have firmly occupied one of the focal spots in mathematics, and make the foundation for some parts of theoretical as well as of applied mathematics. In any case, being an ideal mathematical idea, in practice individuals normally need to bargain not with common, or crisp, sets, that is with the sets as they are grasped in mathematics, yet with set like combinations having dubious, non-sharp or imprecise boundaries. Only a couple of guides to illustrate this assertion. Consider the arrangement of all citizens of some state, say Lithuania. It is a genuine set, a subset with sharp boundaries of the arrangement surprisingly on the planet. Each individual either has a place with this set or not. Then again consider "the set" of youthful Lithuanian individuals. Obviously, this is not a set in the precise mathematical sense: there is no regular age line after which a man or a lady stops to be youthful and becomes non-youthful. In addition, if for certain reasons, for example on the off chance that a competition of applicants for research awards among youthful scientists is declared and according to regulations an individual is youthful if he/she is more youthful than 30 years, this division into youthful and non-youngsters is artificial and even might bring on some issues for a committee awarding such awards. Say if today is the applicant's 30th birthday and just today he has submitted the venture - whether the committee, working officially, considers this application as valid? At any rate the arrangement of youngsters is a not a set yet a "set-like combination", or "a fluffy set" and this combination has non-sharp, unclear, imprecise lines.. In mathematics and its applications one has frequently to manage such sets as, say, the arrangement of all genuine numbers bigger than 10. Obviously it is an illustration of a typical common set. Then again quite frequently we need to manage such set-like combinations as "the set" of all genuine numbers which are approximately equivalent to 10, or "the set" of all numbers which are a lot bigger than 10. Obviously, the boundaries of such aggregates are not sharp and they make instances of fluffy sets. Since ancient times most reasoning's in mathematics are grounded on bivalent, or binary, logic. According to this logic each assertion in mathematics ought to be either obvious or bogus. Be that as it may, in practice dealing with different issues one frequently experiences proclamations which are pretty much evident, or consistent with a huge degree, or are unlikely, and so forth. Such proclamations can't be classified and studied in the casings of the classical, or bivalent, logic, yet have a place with the subject of what is called fluffy logic. Consider the assertions "If an and b are genuine negative numbers, then the item a • b is positive" and "If an is bigger than b and n ≥ 10, a, b, n being regular numbers, then, at that point, a n is a lot bigger than b n". Obviously, the first one is a precise mathematical explanation which is valid, while the subsequent assertion could be viewed as a fluffy proclamation which is pretty much evident. Another model: Consider two explanations "If it is raining there are mists in the sky" and "If there are mists in the sky, then possibly it will rain".

The first one is figured out according to the laws of bivalent logic, while the subsequent one can be viewed as a proclamation in fluffy logic. Since the inception of the idea of a fluffy set and laying down the foundations of fluffy logic in 1965 , the interests of numerous specialists in theoretical mathematics as well as in applications of mathematics in different sciences were directed to applying fluffy sets and rules of fluffy logics in their works. Many examination papers regarding this matter were published; a few ordinary meetings dedicated to the issues of "Fluffy Mathematics" were organized. For instance, like clockwork the International Congress of Fluffy Association happens. Since 1979 in Linz, Austria a yearly "Linz Seminar on Fluffy Sets" is organized - each time its program is dedicated to a particular topic of examination. Since 1992 in Slovakia a biannual meeting "Fluffy Sets: Hypothesis and Applications" is held. At present there are a few diaries specializing in publishing works in

"Fluffy Mathematics" and its applications, in particular International Diary of Fluffy Sets and Frameworks (sent off in 1978 and included in the ISI list) and Iranian Diary of Fluffy Frameworks (sent off in 2004 and included in the drawn out ISI list). Likewise different diaries which have a sufficiently wide range of satisfactory examination papers, and the Diary of Mathematical Modeling and Analysis among them, receive manuscripts where mathematical designs on the basis of fluffy sets are studied or applied. This was one reason why the organizing committee of the yearly tenth meeting Mathematical Modeling and Analysis, which was held in Druskeninkai, Lithuania in May 26-30, 2010, has invited the creator of this paper to give a feature talk in which to present the basic ideas and ideas of the hypothesis of fluffy sets as well as to show a few models illustrating the interplay of fluffy sets in the parts of classical mathematics. The present work is an essentially broadened version of my discussion given at the meeting, its main aim is to give an extremely brief introduction into what we call "Mathematics with regards to fluffy sets" for mathematicians who are not professionals in this field but rather who need to have some idea of this area of current mathematics.

Fuzzy Sets and Fuzzy Reasonings: prehistory 1900 - 1965

The comprehension that in the genuine situations discourses doesn't necessarily lead either to valid or to bogus explanations and that there are numerous assertions among valid and misleading, that is proclamations of steady truth was, obviously, not a novel idea in science. As well as not a new was the idea, that not all objects of this present reality in regard of some property can be classified as "white" or "dark" - there can be many objects of different "shadows of dim varieties", that is objects having a given property within a certain degree In particular, such idea was discussed currently in Aristotle's works, see for example. Nonetheless, in the cutting edge times scientists began to express serious interest to the issue of slow truth and the connected issue of having a property within a certain degree just since the finish of the XIXth century. It is likewise worth to take note of, that first individuals to be mentioned in this regard are scientists of expansive scientific interests, scientists referred to both as philosophers and analysts in definite sciences. Charles Peirce (1839-1914 USA), known as chemists, philosopher and mathematician, stated "Logicians have a lot of dismissed the investigation of dubiousness, not suspecting the important part it plays in mathematical idea". Bertrand Arthur William Russel (1872-1970 Extraordinary Britain), an outstanding mathematician, logician, writer and philosopher, discussed these issues in his treatise "An introduction to Mathematical Philosophy" [63]. In 1937 there was published the monograph "Dubiousness: an exercise in logical analysis" by Max Dark (1909 Azerbaijan - 1970 USA), a philosopher and a scientist in the field of quantum mechanics. In this monograph the creator considered "consistency profiles in request to characterize quantities without clear lines which he called "unclear images" and discussed the importance of obscure images for philosophical issues. Dark's ambiguous images are firmly connected with interval-type fluffy numbers described in this paper in Section 4.1. Presumably the first scientist who tried to concentrate on the issue of uncertainty according to the point of view of mathematical logic was Jan Lukasiewicz (Poland 1878 - Ireland 1956). Lukasiewicz fostered another logical framework, in which explanations can be bogus or valid, yet can be likewise obvious/misleading somewhat. In his talk on Walk 7, 1918 at the Warsaw University he declared that his logical framework "is as rational and independent as Aristotle's logic yet which is a lot richer in regulations and formulae." This logical framework is really a three-esteemed instance of what is perceived by Lukasiewicz logic currently, see section 5.2, see likewise Lukasiewicz's paper. Independently of Lukasiewicz and starting from different premises, one more arrangement of many-esteemed logics was discovered by Emil Post (Russia 1897 - USA 1954). Another mathematician whose contribution to the "prehistory" of Fluffy Mathematics, ought to be mentioned, was Karl Menger (Austria 1902 - USA 1985). K. Menger proposed to foster a hypothesis in which the relation ∈ "component has a place with a set" (which is in the basis of the Cantor set hypothesis) is supplanted by the probability of a component belonging to a set. In his paper Probabilistic math, he utilized objects which can be considered as precursors of fluffy points and which he called foggy sets.

Fuzzy sets and fuzzy logics: early history 1965 - 1975

As we tried to show in the previous section, the idea to find a mathematical idea appropriate to describe objects which are not precisely defined as well as to manage proclamations for the validity of which one can't give a monosemantic reply - "yes" or "no", has arisen in progress of numerous scientists during the first 50% of the XXth century. Anyway the credits of founding the hypothesis of fluffy sets (and "inventing" the expression "fluffy set" itself) are given to the teacher of the Berkley University L.A. Zadeh. L.A. Zadeh was brought into the world in 1921, in Baku, at that time the capital of the Soviet Azerbaijan. Later he moved with his folks to Iran (they were the Iranian citizens) where they lived till 1944. In 1944 Zadeh went for studies to the USA. In 1951 he has received specialist degree in electrical engineering, and in 1963 was chosen as the top of the division of electrical engineering at the University of Berkeley. In 1956 he met S. Kleene (1909-1994), an outstanding logician, the creator of the well-known "Introduction to metamathematics". Friendship with Kleene impacted Zadeh. In particular, this influence showed itself in his idea to involve many-esteemed logics in request to describe the behavior of perplexing electrical frameworks. Later this idea has formed into the idea of a fluffy set. The foundations of the hypothesis of fluffy sets were first evolved. This work was trailed by numerous resulting Zadeh's papers in which different parts of the hypothesis of fluffy sets and fluffy logic and related issues were considered.† The inception of fluffy sets by Zadeh did not remain unnoticed: in the following ten years there were published a few important papers considering fluffy sets in theoretical mathematics (Topology: C.L. Chang , B. Hutton, J.A. Goguen; Variable based math: A. Rosenfeld , D.N. Mordenson and D.S. Malik; Measure and integral hypothesis: L.A. Zadeh, M. Sugeno) and in applied sciences (Decision making: R. Bellman and L. Zadeh, framework analysis: C.R. Negoita and D.A. Ralescu, and so on) An outstanding contribution to the field of "Fluffy Mathematics" was finished by J.A. Goguen, who introduced an important and regular generalization of a fluffy set, the supposed a L-fluffy set (where L could be any finished lattice or even a cl-monoid, [8]) in this way essentially enlarging the extent of possible applications of the original idea of a fluffy set and enriching the mathematical theories with regards to fluffy sets. In this period likewise a few works criticizing the entire idea of a fluffy set were published, see for example. We allude the peruser to for the discussion around this criticism.

Fuzzy sets: basic concepts

To grasp the idea of a fuzzy set note that a subset A of a set X can be described by its membership function $\chi_A : X \to \{0,1\}$:

$$\begin{cases} \chi_A(x) = 1 & \text{if } x \in A \\ \chi_A(x) = 0 & \text{if } x \notin A\ . \end{cases}$$

Thus χ_A characterizes the degree to which an element x belongs to a set A. This degree is either 0 or 1. Now, allowing this degree to be also between 0 and 1 we come to Zadeh's concept of a fuzzy subset A of a set X, denoted $A\tilde{\subset}X$ and characterized by its membership function $\mu_A : X \to [0,1]$. The value $\mu_A(x)$ is interpreted as the degree of belongness of a point $x \in X$ to a fuzzy set A.

Taking into account that fluffy sets are "invented" it request to be a more satisfactory contraption for the reasons for practice, and to reflect better the method of human reasonings, one can easily take note of that in certain situations the unit interval [0, 1] as the codomain for membership functions is excessively restricted. We give an illustration of such situations. It concerns the fluffy set B of birds in the arrangement of every living being. Obviously a swallow and a nightingale have a place with B with degree 1, while a hippopotamus and a lion have a place with it with degree 0. Yet, shouldn't something be said about such "birds" as ostrich, penguin and kiwi? Ornithologists concur that every one of them have a few essential, however not all typical characteristics of birds. So we can interpret that a "bird" can have a place with the "fluffy set" B of birds partially, somewhere in the range of 0 and 1. Anyway characteristic bird traits of an ostrich, a penguin and a kiwi are different. Hence one can scarcely decide which of them has a place with B to a bigger degree. Consequently we come to the idea that a membership function of a fluffy set ought to be permitted to acknowledge likewise incomparable qualities, (in this model incomparable qualities characterizing the belongness level of an ostrich, a penguin and a kiwi to the fluffy set B of birds) between 0 (the base level of belongness) and 1 (the maximal level of belongness). This prompts the idea to consider fluffy sets whose membership functions can acknowledge incomparable esteemed. This idea was created by J.A. Goguen and brought about the idea of a L-fluffy set

Namely, let L be a complete lattice whose bottom and top elements are 0 and 1 respectively, see e.g. [8]. Further let X be a set. Then an L-fuzzy subset $A\tilde{\subset}X$ is characterized by its membership function $\mu_A : X \to L$. Note that in case $L = [0,1]$ an L-fuzzy set is just a fuzzy set $A\tilde{\subset}X$ as defined by L.A. Zadeh, and in case $L = \{0,1\}$ we actually redefine a usual subset A of X.

At this point we note that generally L-fuzzy sets are not only more satisfactory for various applications, but also are theoretically more interesting and promising for applications than its special kind, fuzzy sets as they were defined above. On the other hand the work in the context of general L-fuzzy sets is essentially more complicated than in the context of "ordinary" Zadeh's fuzzy sets, that is L-fuzzy sets in the special case when $L = [0,1]$. Therefore in the sequel we shall consider only the case of "ordinary" fuzzy sets.

When working with fuzzy sets a helpful tool is the so called level decomposition of a fuzzy set. Namely, the subsets $A_\alpha := \{x \in X \mid \mu_A(x) \geq \alpha\}$ of the set X are called (non-strict) levels of a fuzzy set $A\tilde{\subset}X$ and the family $\{A_\alpha \mid \alpha \in L\}$ is called the level decomposition or the horizontal representation of the fuzzy set A. One can easily see that $A_0 = X$ and that $\bigcap_{\beta<\alpha} A_\beta = A_\alpha \ \forall \alpha \in]0,1]$. Conversely, starting with a system of sets with such properties a corresponding fuzzy set A can be reconstructed by setting $\mu_A(x) = \sup\{\alpha \mid x \in A_\alpha.\}$ having A_α as its level sets.

Along with non-strict levels, decomposition of a fuzzy set A into a system $\{A^\alpha \mid \alpha \in [0,1[\}$ of strict levels $A^\alpha := \{x \in X \mid \mu_A(x) > \alpha\}$ is often used.

Conclusion

The starting point of interest in mathematical designs with regards to fluffy sets in Latvia was in the middle of 1980's the point at which the first works in fluffy topology were published by the creator of this paper, see for example, and so on. Over decade have passed before a gathering of youthful Latvian scientists - S.Solovjovs, I.Ul¸jane , I.Zvina, et. al. showed interest in the issues of fluffy topology see for example. Since the finish of nineties the scientific interests of our gathering was attracted likewise to the categorical parts of mathematics with regards to fluffy sets. A series of papers by S. Solovjovs, I. Ul¸jane, O. Grigorenko and myself were published, see for example. Later the range of interests of Latvian mathematicians in the field of fluffy sets included likewise the hypothesis of Measure and Integral; and presently there is accomplished some important work in this field by prof. S. Asmuss and her understudies V. Ruˇza and P. Orlovs, see for example. In the wake of participating at the gathering FSTA (Fluffy Sets - Hypothesis and Applications) in Slovakia in 2008, the interest of a few more youthful individuals from the Latvian gathering was attracted to the hypothesis of Fluffy Aggregation Administrators. The most active among them are J. Lebedinska, O. Grigorenko and P. Orlovs. This interest brought about a series of papers, and so on.

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